Learning through argumentation in community with your students

Chepina Rumsey, Jody Guarino and Tiffany Kane describe how argumentation supported nine- and 10-year-old students in developing a deep understanding of mathematics.

Our students can explore and generate powerful ideas about mathematics. "Each and every child must be afforded opportunities to not only feel confident as doers of mathematics but also to experience *joy* and see *beauty* in their mathematical discoveries" (Huinker (ed.), 2020, p. 17). We can both model this for our students and have opportunities to experience joy and beauty ourselves as teachers and learners. You might be wondering what it looks like to model this kind of learning for our students and how to incorporate it in our classrooms. We have been exploring this alongside teachers and are excited to share some of our insights, as we think about learning in community with our students and how that relates to mathematical argumentation. Mathematical argumentation is one way for curious mathematicians to explore and build a deep understanding of mathematics by communicating their ideas, listening to ideas of others, and making connections. Mathematical argumentation is about nurturing students as they: (1) notice and wonder, (2) conjecture, (3) justify, and (4) extend their ideas (Rumsey and Guarino, 2024). We have seen that argumentation provides an opportunity for students to explore and grapple with mathematical content and build a strong understanding, and it also provides a chance for teachers to learn and grow alongside their students (Rumsey and Guarino, 2024). Argumentation is one way to support and nurture the role of teachers as learners. There are different ways to view the role of the teacher, from someone who imparts knowledge to their students, to someone who monitors learning of students, to someone who facilitates the learning of their students, to a more knowledgeable other (Munter, 2014), see green parts of Figure 1. We see an additional role as vital, which we have added in purple to Figure 1: teachers as learners who learn together in community with their students. We want to highlight this idea of argumentation being an opportunity for teachers too, because often we focus on the positive impact on student learning.

In this article we want to provide a classroom example to show how argumentation can enhance conceptual understanding of students and provide a vision for what teachers as learners in community with their students can look like.

Background

During a professional development project we worked with a school district to explore how to engage students in noticing and wondering, conjecturing, justifying, and extending ideas. Nurturing a playful curiosity in the teachers supported their implementation of the instructional strategies and positioned them as life-long learners who have an opportunity to learn with their students. To share insight into some of the student and teacher learning that is possible through argumentation, we tell this story of a classroom lesson through different lenses, specifically the researcher/PD facilitator lens (Chepina and Jody) and the classroom teacher lens (Tiffany).

Context of the lesson

Students in Tiffany's classroom were beginning to extend their knowledge about fractions to adding and subtracting with fractions. We hoped that integrating argumentation into this topic would provide students with opportunities to deeply think about how operations extend to fractions. Earlier in the week students operated with fractions, solving problems with mixed numbers with the same denominators

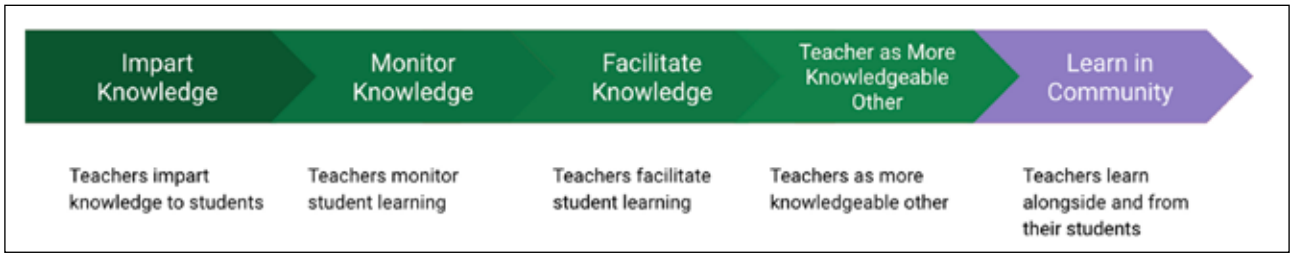

Figure 1: Teacher roles based on Munter, 2014.

such as $4\frac{3}{4} - 2\frac{1}{4}$. For these expressions, there was no need to decompose because the fractional part 4 of the subtrahend is smaller than the fractional part of the minuend. The following day students solved subtraction story problems with mixed numbers. This time the numbers had the fraction of the subtrahend larger than the fraction of the minuend. For example, $4\frac{1}{4} - 2\frac{3}{4}$. Some students realized that they had to revise their thinking with this type of problem because the previous strategies wouldn't work the same. Two students made a connection between decomposing with fractions to decomposing whole numbers using a place value chart that they were familiar with. For example, with whole numbers we can represent 315 as 3 hundreds, 1 ten, and 5 ones and if we were to subtract 245 we can use decomposition to reorganize the numbers and subtract. With fractions, when subtracting $4\frac{1}{4} - 1\frac{3}{4}$, students created a diagram that showed whole number place value regrouping applied to fractions. Figure 2 is the public record used to share this idea with the class (see Figure 2). We share this public record in part because students will refer to it later in this series of lessons as they make sense of the tasks.

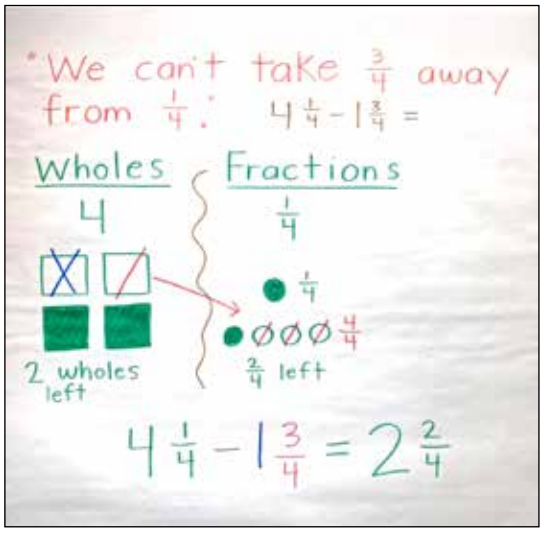

Figure 2: Chart with wholes and fractions.

With these previous activities and student knowledge in mind, we planned the next lesson to include opportunities for students to notice and wonder, make conjectures, justify, and extend so that we could continue to make their thinking, including possible misconceptions, visible to the community. We decided to start with a warmup, using a class routine 'true/false', which can foster mathematical argumentation. When students engage in 'true/false', they are given one equation at a time and asked if each statement is true or false and why. We planned the equations strategically to include equations where the subtrahend is smaller than the fraction in the minuend such as $2\frac{3}{4} - \frac{2}{4} = 2\frac{1}{4}$ and equations displaying an element of decomposition, for example $2 - \frac{3}{4} = 1 + \frac{4}{4} - \frac{3}{4}$.

Following the true/false warm up students were given two related equations (see Figure 3) and several questions to consider, working with a partner or table group. Our goal with the questions was to engage students in noticing and wondering. We also designed the last question to move students toward conjecturing by asking about what might be true more generally.

Show your thinking using drawings, number, or words.

$$3\frac{3}{4} - \frac{1}{4} =$$

$$3\frac{1}{4} - \frac{3}{4} =$$

How are these equations the same and different?

Did you use the same strategy for both equations? If your strategies are different, why did you use different ones?

What do you believe to always be true about subtracting fractions?

Figure 3: Student task.

As they worked on the task, students used strategies that made sense to them, such as those shown on the chart with wholes and fractions (See Figure 2); the number line (See Figure 4), a tape diagram (See Figure 5), and numerical computations (See Figure 6). Their strategies and explanations gave us an insight into how they thought about the subtrahend. We learned by seeing their different strategies and the connections between this task and their prior knowledge. We include samples of the students' work in Figures 4, 5, and 6, which we reproduced for readability.

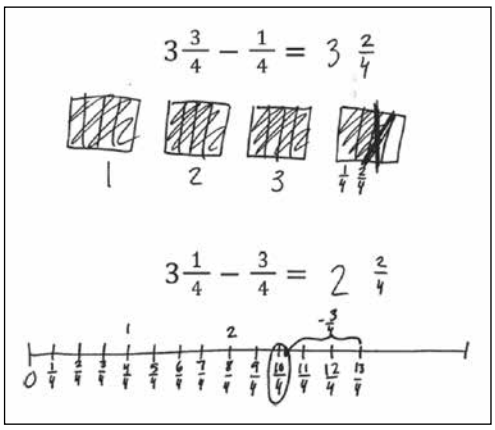

Figure 4: Student strategy with number line.

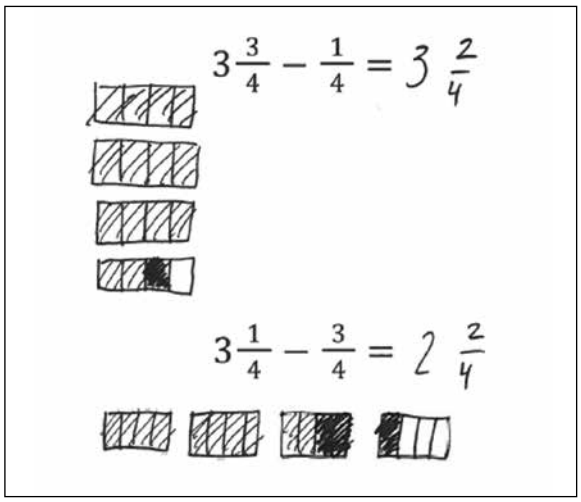

Figure 5: Student strategy with tape diagram.

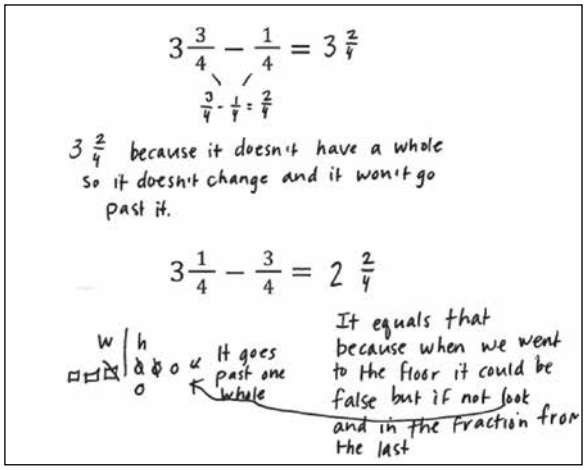

Figure 6: Student strategy with numerical computation.

Some students made sense of how much more is needed to make a whole to help them with the subtraction. Students discovered that there were times when they could subtract without decomposing and times when they needed to decompose. Through collective discussion and making meaning together, students were able to form an initial conjecture that when subtracting fractions, you need to decompose when "the subtraction number is bigger than the one you are trying to subtract." As we worked on precise language, we modified it to say, "You can decompose when the fraction in the subtrahend is larger than the fraction in the minuend." Modifying the conjectures not only supported precise language and deeper connections, it also helped bring everyone into the community of learners by having a shared language that everyone understands. Giving the students a chance to notice, wonder, conjecture, and explain their thinking allowed for deeper understanding of patterns and operating with fractions. The elements of the lesson relating to argumentation supported the students' conceptual understanding but also provided a chance for us to learn and grow alongside them.

Reflecting on student and teacher learning in community

While the students learned about adding and subtracting fractions through the lesson, as the three of us reflected on the lesson, our own learning continued to be a theme in the conversation as we discussed the learning of the students. To give you insight into our conversation, let us hear from the classroom teacher, Tiffany. She reflected that within this work she was learning alongside her students, saying:

> Argumentation was new to me. I was trying things in my classroom I'd never tried before. For example, I thought about what question I would pose to students to nudge them to make conjectures. Sometimes those questions were successful and led to conjecture making, other times they didn't. I was building my own math knowledge and developing instructional tools by experimenting. I learned that students can explore and share conjectures, argumentation helps students build a stronger understanding, and learning can be messy and unpredictable (for teachers and students)! Just as we ask our students to persevere and solve problems, as a teacher, I was doing this too! We have to be patient with ourselves as teachers as we try new things; we're learners too.

We also reflected on the incorporation of mathematical argumentation into what was a challenging topic. Tiffany noted that:

> by providing opportunities and time for students to think about these types of fraction problems, I learned that this is crucial for my students to really make sense of the work, eventually leading them to form a conjecture. Throughout this year, my learning of argumentation has shifted my teaching. The students were leading these ideas and making meaning together.

Argumentation in the context of fractions supported deep and meaningful learning as students noticed, wondered, conjectured, and justified their ideas. It also supported deep and meaningful learning by the adults in the room. For example, Jody's classroom observation notes show how she was thinking about the lesson and engaging with student thinking (see Figure 7).

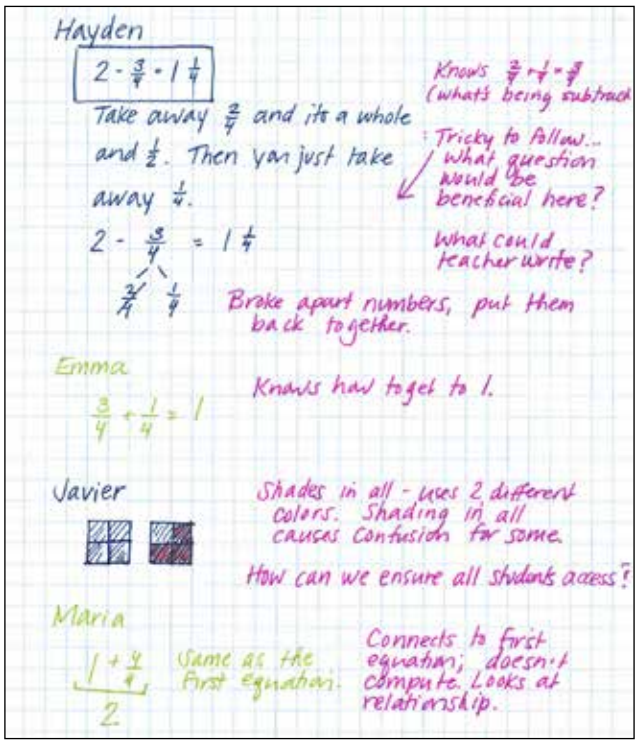

Figure 7: Jody's notes from the lesson.

As we reflected on the student and teacher learning that occurred in the classroom as everyone learned together in community, we wondered what attitudes supported us, especially Tiffany. Specifically, we wondered what beliefs she has and how she thinks about teaching and learning. We note that there is a respectful agency at the school where teachers are encouraged to use their professional knowledge to create lessons that support student learning. That creative agency also extends to the students in this classroom. Tiffany positions herself as a learner and is naturally curious. As evidenced in her statement above, she realizes that messiness is okay and part of learning. As we were planning the lesson a couple of times she said, "I just want to learn and see what they do." She supports an environment where students engage, explore, and share in the classroom. Because Tiffany is willing to try new things, the students were also using that model to confidently explore. She models what it means to learn for her students.

Conclusion

As we step back and think about our work as teachers, we have a unique opportunity to learn from the work of teaching. Argumentation provides us with a context where both teachers and students can become a community of learners and we can learn with and from each other. For example, as we (teachers) think about what students understand, how to leverage their ideas, or consider questions we might pose, these reflections lead us to hypothesize, and experiment, adding to our repertoire. We all can grow in our ability to notice and make sense of the world and through argumentation we can do that along with our students. The idea that including mathematical argumentation can have benefits for students and our own learning has been influential to us and we have been eager to share this with others. We can learn in community with our students when we model curiosity, ask questions, and share our questions and wonderings with our students.

Chepina Rumsey, PhD, is an associate professor of mathematics education at the University of Northern Iowa in Cedar Falls, Iowa and **Jody Guarino,** EdD, is a lecturer in the School of Education at the University of California, Irvine. Chepina and Jody have a forthcoming book, Nurturing Math Curiosity (Solution Tree, May 2024) connecting to ideas in this article**. Tiffany Kane** is a classroom teacher in a southern California school district.

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