MATHEMATICS TEACHER
Learning & Teaching
PK 12

Mathematics Teacher: Learning and Teaching PK-12, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

Subtraction, Decomposition, and Argumentation

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DIGITAL OBJECT IDENTIFIER: 10.5951/MTLT.2022.0168

VOLUME: 116

ISSUE NUMBER: 2

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Approved by the NCTM Board of Directors on July 15, 2017.

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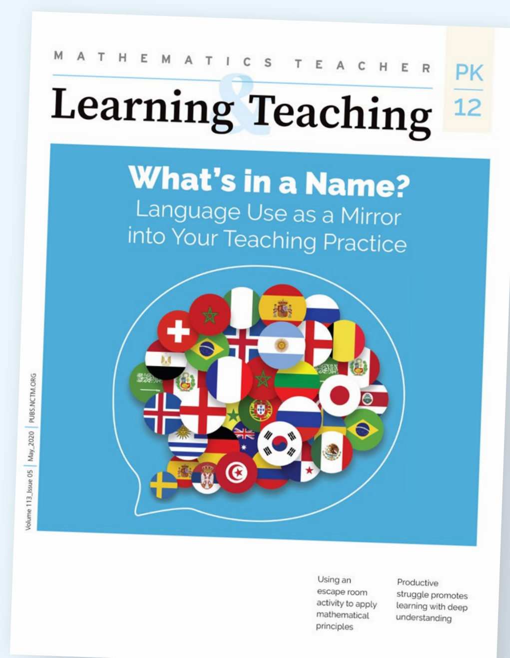

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Subtraction, Decomposition, and Argumentation

We describe how mathematical argumentation supports curiosity and exploration by sharing a first-grade lesson in which students explored decomposition with subtraction. We also reflect on the conditions that supported the inclusion of mathematical argumentation.

Chepina Rumsey, Jody Guarino, and Michelle Sperling

What would it look like to leverage students' natural curiosity during mathematics class so that they can develop an understanding that is built on what they notice and wonder? How might we organize an exploratory experience to teach mathematical content like procedural fluency and subtraction facts? One way that we have been intentional about including spaces for curiosity and exploration is by integrating mathematical argumentation. This supports the development of students' conceptual understanding as they explore and tinker with the mathematical content (Rumsey & Langrall, 2016).

Mathematical argumentation is about exploring, conjecturing, justifying, and sharing ideas. We have found that considering these parts, or layers, of argumentation is a helpful way to think about what it means to argue mathematically, and thus, infuse

“Children enter this world as emergent mathematicians, naturally curious, and trying to make sense of their mathematical environment” (NCTM, 2020, p. 17).

curiosity and exploration. We considered these layers on the basis of our work from two separate year-long quality-improvement initiatives on the west coast of the United States (Rumsey et al., 2019). The word *layers* emerged in our work because it was helpful for teachers to see how the components of argumentation build on each other. Once one “layer” is introduced, another can be built on top of that foundation. We describe the Four Layers of Argumentation in Table 1 (also see supplementary handout for Quick Reference Questions [link online]).

To illustrate an exploration of mathematical ideas within the layers of argumentation, we present an example of a two-lesson series from a first-grade classroom in the context of subtraction. The first-grade teacher, Michelle Sperling, was a participant in a quality-improvement initiative in the district and a related professional development series. We describe the lessons that took place in her classroom and reflect on what the conditions were that supported the mathematical argumentation to be successful.

STORY OF GRADE 1 LESSONS

Prior to this lesson sequence, students had solved two-digit addition problems like 34 + 57. Most students regularly used place value strategies such as “combining like units” (adding tens 30 + 50 to get 80 and then adding ones 4 + 7 to get 11, and then combining tens and ones to get a total of 91). Some students used a variation of that for a multiple of 10; having the numbers 80 and 11 to combine, they decomposed 11 to 10 and 1, then added 10 to the 80 to get 90 and then the additional 1 to get 91. Students in the first-grade classroom had been successful in solving problems like this; using place value; and representing their thinking with pictures, diagrams, and equations.

We wondered how students’ understanding of numbers, place value, and addition would transfer to subtraction. We decided to narrow our focus to subtraction strategies and invite students to consider when a particular strategy is useful. An important goal is for students to “build procedural fluency from conceptual understanding” (NCTM, 2014), so we wanted students to consider—on the basis of their conceptual understanding—when to flexibly use a procedure. We also wanted to provide an opportunity related to subtraction for students to notice and wonder, conjecture, justify, and share ideas.

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doi:10.5951/MTLT.2022.0168

Students had recently worked with expressions like 14 – 5 and 15 – 7 and were building procedural fluency by exploring different strategies, including counting back, drawing and crossing off circles, and directly modeling on a ten-frame; a few students decomposed the subtrahend (see Figure 1).

Building on what students had been working on, we had the following lesson goals:

- Intentionally position students as competent mathematicians.
- Create opportunities for students to explore, while we as teachers curiously learned alongside the students.
- Build procedural fluency as the students grapple with when a strategy is useful.

Day 1

The teacher started by writing the expression 17 – 8, which was selected because the subtrahend (8) is bigger than the digit in the ones place of the minuend (7). The hope was that the numbers would lead some students to decompose and create space for the discussion about when to use a particular strategy, in our case, to decompose. As students worked to find the value of the expression (see supplementary

Figure 1 Anchor Chart From Student Share Out of 14 – 5

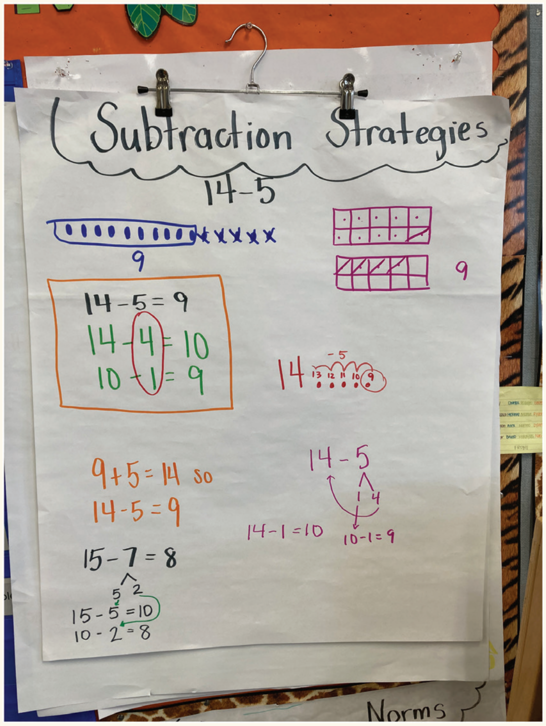


Table 1 Four Layers of Argumentation

Layer	Description	Questions to Foster This Layer
Notice and wonder	• Explore and observe patterns. • Collect information about the patterns. • Make connections between examples. • Ask curious questions.	• What do you notice? • What do the equations have in common? • Who has a related observation? • What do you wonder?
Conjecture	• Consider how the pattern will extend. • Extend a specific observation to a generalization about all numbers or cases.	• Is that observation always true? • What do you believe to always be true about ___? • When does the strategy work? • Is that true always, sometimes, or never? How do you know?
Justify	• Convince someone that this idea always works. • Explain the conjecture and why it works.	• How could you convince someone that your conjecture is always true? • Why is that true?
Share and modify	• Share the idea with others. • Modify and refine the conjecture and justification through a discussion with classmates. • Include precise language.	• Does anyone have a similar idea or something you'd like to add? • Does anyone have a different idea? • How can we rewrite our conjectures to make them more precise?

handout [link online]), teachers walked around using a monitoring tool (Stein et al., 2008) to notice the student strategies, purposefully looking for students who used a ten-frame and for students who decomposed 8 into 7 and 1 (see supplementary Monitoring Chart [link online]). We looked for these representations because we wanted to highlight them in the next part of the lesson.

After debriefing about the selected strategies as a whole group, students were then given a second task (see Figure 2 and supplementary handout [link online]) in which they were invited to explore and make observations about three expressions, each with the subtrahend larger than the ones place of the minuend, launching the first phase of argumentation (see Table 1).

Students worked in small groups. As we monitored them, we heard conversations including these remarks:

- "There's more than one way to get to nine."
- "Decomposing is useful sometimes if the second number [of the expression] is big."
- "If the second number is small, you don't need to decompose."

After having time to explore and discuss in small groups, students returned to the whole group and shared their ideas about what they noticed and wondered (Argumentation Layer 1, see Table 1) and what they believed to be true (Argumentation Layer 2, see Table 1). Building from their noticings and wonderings, some students were beginning to develop conjectures (see Table 2), and some provided additional expressions they considered with their partners. On the anchor chart (see Figure 3), the teacher wrote the observations, additional student ideas, and the working conjectures: (a) If the second number is big, decompose; (b) if the second number is small, you don't need to decompose. The students

Figure 2 Task to Support Early Conjectures Through Notice and Wonder

What observations can you make about these expressions?

What do you notice and wonder?

14 – 5   15 – 6   15 – 7

Table 2 The Layers Unfolding Within Day 1

Layer	Day 1 Examples of Responses
Notice and wonder	• 10 plus 7 equals 17, we can show that on the ten-frames. • **For 17 – 8, there aren't enough circles to cross off in the second ten-frame.** • There are different ways to break up 8 (for example 7 and 1, 6 and 2). • **You can subtract in parts and still get the same answer.** • **You can subtract to make a "friendly 10" and then subtract more.** • There is a 7 and 1 in both strategies. • We can use the same equation for different strategies. • More than one way to get 9. • **Decomposing is useful, sometimes.**
Conjecture	• There is more than one way to get 9 as the answer. • **If the second number is big, decompose.** • **If the second number is small, you don't need to decompose.**
Justify	
Share and modify	

Note. Bolded ideas were leveraged to advance the discussion. Colored pairings make visible how the conjectures were revised.

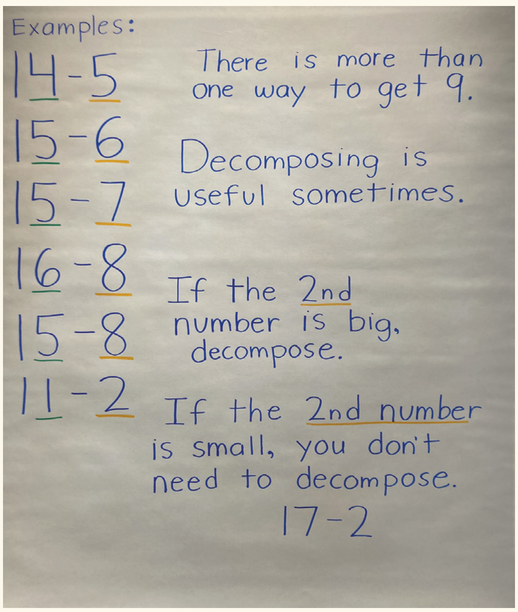
had used the word *conjecture* in context in their classroom since the fall and had a shared understanding that conjectures are statements you believe to be true that we can continue to explore. The lesson ended with the sharing and recording on the anchor chart.

After the Day 1 lesson, we reflected as a group and considered the student's statement regarding whether "the second number is big or small." We started to wonder, "What makes a number big? What makes a number small?" We also thought about the phrase *second number* and how we could support the students to use more precise language. As a result, we designed a task to use on Day 2 that included the Day 1 conjectures and would create an opportunity to grapple with those questions and refine the shared language (see Figure 4).

Day 2

We began Day 2 by revisiting Day 1 conjectures and asking students, "Is five big or small?" Expressions were organized in two columns, one in which decomposing was not useful and the other when it was. We decided to include ten-frames to support sense making by making it visible when you could remove dots from only the second ten-frame and when you could also remove dots from the 10 (decomposing the subtrahend).

Figure 3 Anchor Chart After Day 1



Students saw patterns and used the task to justify their thinking and share and modify their conjectures (see Table 3). Within the table, we have included a summary of student responses, organized by the corresponding layer of argumentation. As student ideas were shared and recorded, the teacher decided which ideas to leverage on the basis of the lesson goals. Bolded ideas were leveraged to advance the discussion. Colored pairings make visible how the conjectures were revised.

As a whole group, the students shared ideas that emerged in their groups. A student shared that decomposing is useful sometimes. Another student offered that if the second number is big, decompose. A third student added that if the second number is small, you do not need to decompose. This led to the need for clarification because the language is vague and imprecise. When the language *second number* was used, what did that refer to? In the example of 15 – 8, was the "second number" the digit 5 in 15 or the 8? The student clarified that he meant the 8, leading the teacher to introduce the terms *subtrahend* and *minuend* (see Figure 5). Precise language aided in communication and was added to the chart in a different color to make it more apparent.

Students had (a) noticed and wondered and (b) made conjectures, the first two layers of mathematical argumentation. Next, they were ready to begin to justify, share, and modify their conjecture. The teacher selected one problem from each column on the student sheet, added a visual representation, and asked "Is five big or small?" One student responded, "Five is small because when you count, you only cross off from one ten-frame," and another said, "It depends." A third student noticed that "five is big because there aren't enough ones. You cross off from the full ten-frame." The teacher then asked: "What does it depend on? How do we know if it's considered big? How do we know if it's considered small?"

Because we had intentionally selected five as the common subtrahend for each expression and because of the visual representation of the ten-frames, students focused on the relative size of five compared to the minuend's ones place, or the ones that were beyond the 10 of the teen number. This led to modifying the conjectures as the teacher asked: "How can we rewrite our conjectures to make them more precise?" Figure 6 shows students' final conjecture below the ten-frames. As the final conjecture was charted, the teacher revoiced the conjecture and asked students to generate expressions that

Figure 4 Student Task to Address Class Conjectures From Day 1

Name: ___________________

Our Conjectures:

- If the second number is big you decompose.
- If the second number is small you don't need to decompose.

Is 5 big or small?

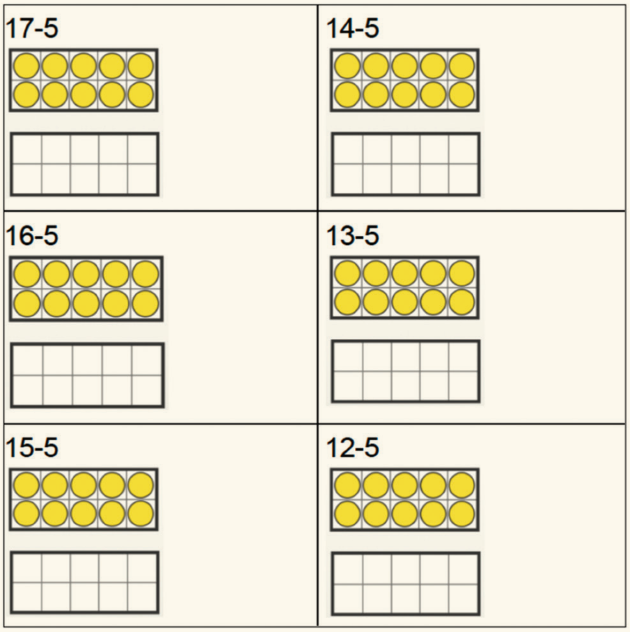


Table 3 The Layers Unfolding Within Day 2

Layer	Day 2 Examples of Responses
Notice and wonder	• **Five is "big" sometimes and "small" sometimes.** • Arrows show the connection between representations.
Conjecture	• If the second number is big, decompose. • If the second number is small, you don't need to decompose.
Justify	• **Ten-frames show crossing off in the first ten-frame for "big" numbers.** • **Ten-frames show that you don't need to cross off from the first ten-frame for "small" numbers.** • Five is small because when you count, you only cross off from one ten-frame. • Five is big because there aren't enough ones. You cross off from the full ten-frame.
Share and modify	The pair of conjectures was modified three times during the group discussion to make the language more precise: • If the second number is big, decompose. • If the second number is small, you don't need to decompose. • If the subtrahend is big, decompose. • If the subtrahend is small, you don't need to decompose. • If the subtrahend is bigger than the ones place of the minuend, decomposing is helpful. • If the subtrahend is smaller than the ones place of the minuend, decomposing is not needed.

Note. Colored pairings make visible how the conjectures were revised, organized by the corresponding layer of argumentation.

matched each conjecture. Students offered additional expressions that were then written on sticky notes and added to the chart. This final step of the lesson was helpful because students were then able to make sense of and generate specific cases to build their understanding. It allowed the teacher a quick formative assessment to gauge student understanding and gave students an opportunity to take ownership of the conjecture.

Figure 5 Anchor Chart After Day 2

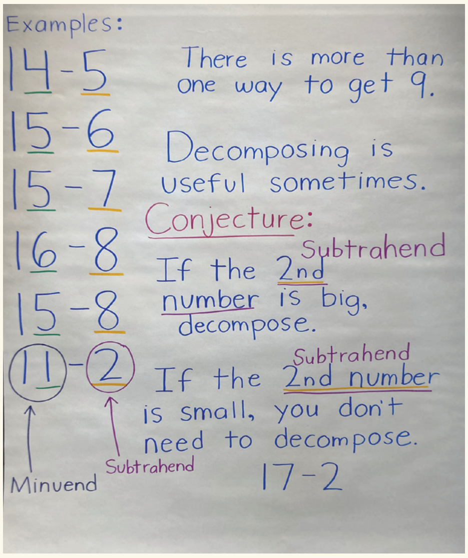
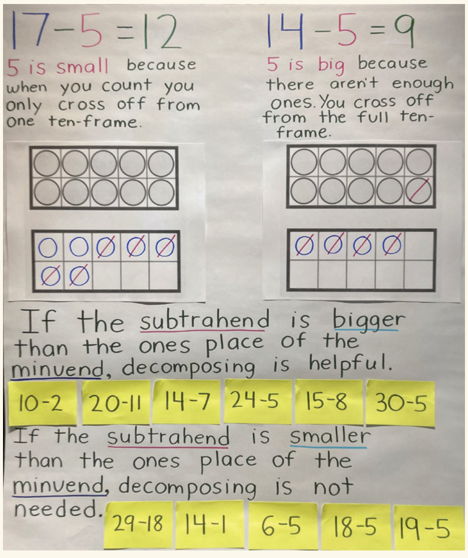


Figure 6 Final Conjecture Chart



Conditions Instrumental in Implementing Argumentation

As we reflected on the instructional sequence above, we want to draw attention to the conditions within the task design and lesson enactment that we believe were instrumental to the student learning experience. In Table 4 we include each condition along with a specific example from the lessons and what this afforded teachers and students.

GETTING STARTED

Although the lesson we describe in this article addresses decomposition of numbers when subtracting, other big ideas of subtraction would integrate well with argumentation. For example, students might explore what happens when 10, or multiple 10s, are subtracted from a number. They may notice that the digit in the ones place does not change and might explore why. As they work with increasing quantities, they may see similarities when subtracting 100 or multiple 100s. These understandings are important in primary grades and later will be built upon as students think about subtraction with other types of numbers including fractions, decimals, and integers. The focus of our work was subtraction, but argumentation can be grounded in any important mathematical ideas (Russo, 2018).

As you explore argumentation with your students in the context of subtraction, here are four ways you might get started (see Table 5 for specific K–2 examples):

- Big Ideas: Consider big ideas specific to the content you are working on. Anticipate what students may notice, wonder, and conjecture about.
- Open-Ended Task: Give students an open-ended task that will provide multiple examples to analyze (see Figure 2, Figure 4, and supplementary handouts [link online]), allowing students to generalize across examples.
- Pose Questions: Pose questions that will encourage students to notice patterns and think about what might be true on the basis of those noticings (see supplementary handout [link online] and Thoughtful Questions within Table 4 for ideas).
- Test and Share Conjectures: Provide opportunities for students to test the conjectures and share their thinking within the community.

Table 4 Conditions Instrumental in Implementing Argumentation

Condition	Example From Lesson	What This Afforded
Task Design Phase		
Intentional number choice	Problem sets that have some answers less than 10 and some more than 10	The chance to compare problems and notice when decomposing is a good option
Making the structure of mathematics visible through representations	Including double ten-frames on the task sheet Numbers placed in sequential order in two columns (see Figure 4)	Students visualize the quantity being removed and remaining quantity. Students could notice number patterns.
Generalizing across expressions	To answer the question "Is five big or small?" students were provided with several expressions in sequential order to use as a reference (see Figure 4).	Multiple expressions allowed students to see and make sense of patterns across cases.
Lesson Enactment Phase		
Thoughtful questions	• When is the decomposing strategy useful? • Does the strategy always work? • What other numbers would it work for?	Questions nudged students to analyze a strategy, generalize, and consider when the strategy is useful.
Strategic color coding	Colors were used in charting to add clarity to academic language and quantities (see Figures 5 and 6).	A visual opportunity to make meaning and connect language and quantities
Precise language the class has a shared understanding about	Introducing precise language for the generalization (see Figure 5)	Access communication tools beyond the classroom or school as students develop language of the discipline
Examples to help make the conjecture clearer/more concrete	Students shared examples of expressions that fit the conjecture (see Figure 6).	Students made sense of the conjecture, and the teacher had insight into specific student understanding.

Table 5 Four Ways to Get Started

	Kindergarten	Grade 1	Grade 2
Possible big ideas for subtraction	Teen number minus 10	Subtracting 10 (or other decade numbers)	Subtracting 100
Open-ended task, examples to analyze	15 – 10 14 – 10 13 – 10 12 – 10 11 – 10	29 – 10 29 – 20 28 – 10 28 – 20 27 – 10 27 – 20 26 – 10 26 – 20	125 – 100 225 – 100 325 – 100 425 – 100
Possible questions	What patterns do you notice? What do you wonder? What is happening in this collection of expressions? What other expressions would fit into this collection? How are the answers related to the minuend and subtrahend? What do you believe to be true about subtracting ___ (10 or 100)?		
Test and share conjectures	When is this conjecture true? What other examples can we try? How can we modify the conjecture so that it is more precise?		

CONCLUSION

It is possible to include exploratory experiences in mathematics lessons, which not only deepens the understanding of the mathematical content, but also emphasizes practices like mathematical argumentation. Even topics related to procedural fluency and subtraction facts can be adapted to engage students in argumentation. Building on students' natural curiosity, the Four Layers of Argumentation is a helpful framework for incorporating it into lessons (Rumsey & Guarino, forthcoming book 2024). We have documented an example of what occurred in a first-grade lesson when students explored the decomposing strategy to support a conceptual understanding over a 2-day lesson sequence. The students (a) noticed and wondered, (b) conjectured, (c) justified using representations, and (d) shared and modified their ideas so that the conjectures were revised three times. We hope that the Layers of Argumentation, conditions we found instrumental, and additional subtraction topics are useful as you curiously explore with your own students!

REFERENCES

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.*

National Council of Teachers of Mathematics. (2020). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations.*

Rumsey, C., & Langrall, C. (2016). Promoting mathematical argumentation. *Teaching Children Mathematics, 22*(7), 413–419. https://doi.org/10.5951/teacchilmath.22.7.0412

Rumsey, C., & Guarino, J. (Forthcoming 2024). *Let's grow math wonder in young mathematicians* [working title]. Norton.

Rumsey, C., Guarino, J., Adams, R., Cho, C., & Lockhart, B. (2019). Tools to support K–2 students in mathematical argumentation. *Teaching Children Mathematics, 25*(4), 208–217. https://doi.org/10.5951/teacchilmath.25.4.0208

Russo, J. (2018). Mathematical proofs in primary schools. *Mathematics Teaching, 261,* 32–35.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Williams, H. J. (2022). Playful mathematics. *Mathematics Teaching, 283,* 8–10.

